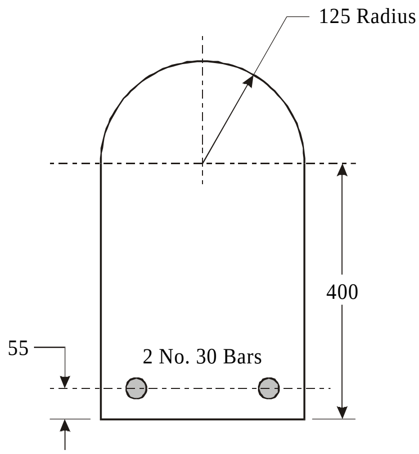
Question 1(a):



$b := 250 \cdot mm$ $h := 400 \cdot mm$ $cc := 55 \cdot mm$

$f'_c := 25 \cdot MPa$ $\varepsilon_{cu} := 0.0035$

$E_s := 200000 \cdot MPa$ $f_y := 400 \cdot MPa$

$\varepsilon_y := \frac{f_y}{E_s}$ $\varepsilon_y = 0.002$ $A_{bar} := 700 \cdot mm^2$

$A_s := 2 \cdot A_{bar}$ $A_s = 1400\,mm^2$

- Effective Depth: $d := h + \frac{b}{2} - cc$ $d = 470\,mm$

$\phi_c := 0.6$ $\phi_s := 0.85$

Solution:

- For $f'_c = 25\,MPa$ $\alpha_1 = 0.813$ $\beta_1 = 0.907$

- Circular section: $r := \frac{b}{2}$ $r = 125\,mm$ $A_1 := \frac{\pi \cdot r^2}{2}$ $A_1 = 24.544 \times 10^3\,mm^2$

$c_1 := \frac{4 \cdot r}{3 \cdot \pi}$ $c_1 = 53.1\,mm$

- Assume compressive stress block extends into rectangular section (a>125)

$C_{c1} := (\phi_c \cdot \alpha_1 \cdot f'_c) \cdot A_1$ $C_{c1} = 299.1\,kN$

- Area of tensile steel to balance semi-circle: Assume steel yields

$C_{c1} = T_1$ $(\phi_c \cdot \alpha_1 \cdot f'_c) \cdot A_1 = \phi_s \cdot f_y \cdot A_{s1}$

$A_{s1} := \phi_c \cdot \alpha_1 \cdot f'_c \cdot \frac{A_1}{(\phi_s \cdot f_y)}$ $A_{s1} = 879.8\,mm^2$

- Remaining area:

$A_{s2} := A_s - A_{s1}$ $A_{s2} = 520.2\,mm^2$

- Location of neutral axis: Define "a_2" = Distance stress block extends below semi-circle

$\Sigma F_x = 0$ $C_{c2} = T_2$ $(\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (a_2 \cdot b) = \phi_s \cdot f_y \cdot A_{s2}$

$$a_2 := \phi_s \cdot f_y \cdot \frac{A_{s2}}{(\phi_c \cdot \alpha_1 \cdot f'_c \cdot b)} \qquad a_2 = 58.1\,\text{mm}$$

Total depth of stress block: $a := r + a_2$ $\qquad a = 183.1\,\text{mm}$

$$c := \frac{a}{\beta_1} \qquad c = 201.7\,\text{mm}$$

Check: $\varepsilon_s := \frac{d - c}{c} \cdot \varepsilon_{cu}$ $\qquad \varepsilon_s = 0.00466$ $\qquad > \varepsilon_y$ OK

or $\frac{c}{d} = 0.4$ $\quad < \quad$ $\dfrac{700}{700 + \dfrac{f_y}{MPa}} = 0.6$ $\qquad$ OK

Moment resistance from semi-circle: Moment arm = $d - r + c_1 = 398.1\,\text{mm}$

$$M_{r1} := \phi_s \cdot f_y \cdot A_{s1} \cdot (d - r + c_1) \qquad M_{r1} = 119.1\,\text{kN}\cdot\text{m}$$

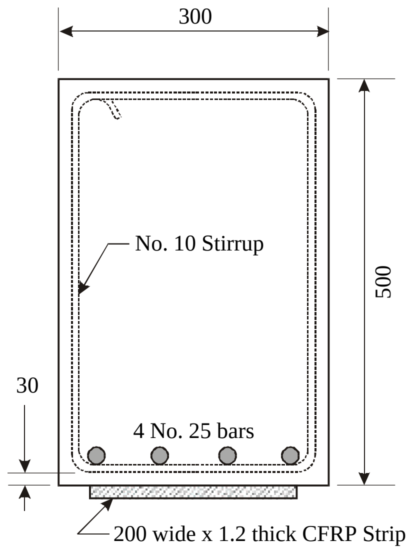
Moment resistance from rectangular portion: Moment arm = $d - r - \frac{a_2}{2} = 316\,\text{mm}$

$$M_{r2} := \phi_s \cdot f_y \cdot A_{s2} \cdot \left(d - r - \frac{a_2}{2}\right) \qquad M_{r2} = 55.9\,\text{kN}\cdot\text{m}$$

Total moment resistance:

$$M_r := M_{r1} + M_{r2} \qquad M_r = 175\,\text{kN}\cdot\text{m}$$

Question 2:



$b := 300 \cdot mm$ $h := 500 \cdot mm$ $cc := 30 \cdot mm$

$f'_c := 30 \cdot MPa$ $\varepsilon_{cu} := 0.0035$

$E_s := 200000 \cdot MPa$ $f_y := 350 \cdot MPa$

$\varepsilon_y := \frac{f_y}{E_s}$ $\varepsilon_y = 0.00175$

Tension steel: $A_{bar} := 500 \cdot mm^2$ $d_b := 25 \cdot mm$

$n_{bar} := 4$

$A_s := n_{bar} \cdot A_{bar}$ $A_s = 2000\, mm^2$

CFRP strips: $b_{CFRP} := 200 \cdot mm$ $t_{CFRP} := 1.2 \cdot mm$ $E_{CFRP} := 165000 \cdot MPa$

$A_{CFRP} := b_{CFRP} \cdot t_{CFRP}$ $A_{CFRP} = 240\, mm^2$ $\phi_{CFRP} := 0.7$

Solution:

- For $f'_c = 30\, MPa$ $\alpha_1 = 0.805$ $\beta_1 = 0.895$

- Effective Depths: $d := h - (cc + 10 \cdot mm) - \frac{d_b}{2}$ $d = 447.5\, mm$

$d_{CFRP} := h + \frac{t_{CFRP}}{2}$ $d_{CFRP} = 500.6\, mm$

Location of the neutral axis: $\Sigma F_x = 0$

$C_c = (\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (\beta_1 \cdot c \cdot b)$ $T = \phi_s \cdot f_y \cdot A_s$

$T_{CFRP} = \phi_{CFRP} \cdot f_{CFRP} \cdot A_{CFRP} = \phi_{CFRP} \cdot (\varepsilon_{CFRP} \cdot E_{CFRP}) \cdot A_{CFRP}$

Strain in CFRP: $\varepsilon_{CFRP} := \varepsilon_{cu} \cdot \left(\frac{d_{CFRP} - c}{c} \right)$

$$(\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (\beta_1 \cdot c \cdot b) = \phi_s \cdot f_y \cdot A_s + \phi_{CFRP} \cdot \left[\varepsilon_{cu} \cdot \left(\frac{d_{CFRP} - c}{c} \right) \cdot E_{CFRP} \right] \cdot A_{CFRP}$$

Rearranging:

$A \cdot c^2 + B \cdot c + C = 0$ where:

$A := (\phi_c \cdot \alpha_1 \cdot f'_c \cdot \beta_1 \cdot b)$ $A = 3890.6 \frac{N}{mm}$

$B := -\phi_s \cdot f_y \cdot A_s + \phi_{CFRP} \cdot \varepsilon_{cu} \cdot E_{CFRP} \cdot A_{CFRP}$ $B = -497.98 \times 10^3 N$

$C := -\phi_{CFRP} \cdot \varepsilon_{cu} \cdot d_{CFRP} \cdot E_{CFRP} \cdot A_{CFRP}$ $C = -48.568 \times 10^6 N \cdot mm$

Solving for "c"

$c = 192.8 mm$ $a := \beta_1 \cdot c$ $a = 172.5 mm$

Check Equilibrium: $C_c := (\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (\beta_1 \cdot c \cdot b)$ $C_c = 749.9 kN$

$T := \phi_s \cdot f_y \cdot A_s$ $T = 595 kN$

$T_{CFRP} := \phi_{CFRP} \cdot \left[\varepsilon_{cu} \cdot \left(\frac{d_{CFRP} - c}{c}\right) \cdot E_{CFRP}\right] \cdot A_{CFRP}$ $T_{CFRP} = 154.9 kN$

$C_c - T - T_{CFRP} = 0 kN$ - OK

Check yielding:

$\varepsilon_s := \varepsilon_{cu} \cdot \left(\frac{d - c}{c}\right)$ $\varepsilon_s = 0.0046$ $\frac{\varepsilon_s}{\varepsilon_y} = 2.6$ OK - Tension steel yields

Stress in CFRP: $\varepsilon_{CFRP} := \varepsilon_{cu} \cdot \left(\frac{d_{CFRP} - c}{c}\right)$ $\varepsilon_{CFRP} = 0.00559$

$f_{CFRP} := \varepsilon_{CFRP} \cdot E_{CFRP}$ $f_{CFRP} = 922.28 MPa$ OK - less than strength

Moment resistance: Take moments about C_c

Moment resistance from steel: $M_T := \phi_s \cdot f_y \cdot A_s \cdot \left(d - \frac{a}{2}\right)$ $M_T = 214.9 kN \cdot m$

Moment resistance from CFRP: $M_{CFRP} := \phi_{CFRP} \cdot f_{CFRP} \cdot A_{CFRP} \cdot \left(d_{CFRP} - \frac{a}{2}\right)$

$M_{CFRP} = 64.2 kN \cdot m$

Total moment resistance: $M_{tot} := M_T + M_{CFRP}$ $M_{tot} = 279.1 kN \cdot m$

Question 3:

Given: $f'_c := 30 \cdot MPa$ $\quad q_L := 3.6 \cdot kPa$ $\quad l_{nb} := 7.5 \cdot m$ $\quad b_{sup} := 400 \cdot mm$ $\quad f_y := 400 \cdot MPa$

$L_{slab} := 4.5 \cdot m$ $\quad b_w := 250 \cdot mm$ $\quad h_s := 200 \cdot mm$ $\quad h_b := 450 \cdot mm$ $\quad \varepsilon_y := \frac{f_y}{E_s}$

$$\alpha_1 := \left| \begin{array}{l} 0.85 - 0.0015 \cdot \frac{f'_c}{MPa} \text{ if } 0.85 - 0.0015 \cdot \frac{f'_c}{MPa} \ge 0.67 \\ 0.67 \text{ otherwise} \end{array} \right. \qquad \alpha_1 = 0.805$$

$$\beta_1 := \left| \begin{array}{l} 0.97 - 0.0025 \cdot \frac{f'_c}{MPa} \text{ if } 0.97 - 0.0025 \cdot \frac{f'_c}{MPa} \ge 0.67 \\ 0.67 \text{ otherwise} \end{array} \right. \qquad \beta_1 = 0.895$$

Effective flange width: $l_{n\_slab} := L_{slab} - b_w$ $\quad l_{n\_slab} = 4250\,mm$

$$b_{LF} := \left| \begin{array}{l} b_1 \leftarrow \frac{l_{nb}}{12} \text{ if } \frac{l_{nb}}{12} \le 6 \cdot h_s \\ b_1 \leftarrow 6 \cdot h_s \text{ otherwise} \\ b_2 \leftarrow b_1 \text{ if } b_1 \le \frac{l_{n\_slab}}{2} \\ b_2 \leftarrow \frac{l_{n\_slab}}{2} \text{ otherwise} \\ b_2 \end{array} \right.$$

$\frac{l_{nb}}{12} = 625\,mm$ $\quad 6 \cdot h_s = 1200\,mm$

$\frac{l_{n\_slab}}{2} = 2125\,mm$

$b_{LF} = 625\,mm$

$b_{eff} := b_w + b_{LF}$ $\quad b_{eff} = 875\,mm$

Loading from slab: $b_{slab} := 1.0 \cdot m$ $\quad \gamma_c := 2400 \cdot \frac{kg}{m^3}$ $\quad \alpha_D := 1.25$ $\quad \alpha_L := 1.5$

Slab self weight: $w_{Ds} := (h_s \cdot b_{slab}) \cdot (\gamma_c \cdot g)$ $\quad w_{Ds} = 4.71 \frac{kN}{m}$

$w_{Df} := \alpha_D \cdot w_{Ds}$ $\quad w_{Df} = 5.88 \frac{kN}{m}$

Live load: $w_L := q_L \cdot b_{slab}$ $\quad w_L = 3.6 \frac{kN}{m}$ $\quad w_{Lf} := \alpha_L \cdot w_L$ $\quad w_{Lf} = 5.4 \frac{kN}{m}$

Reaction on edge beam: $R_f := 0.375 \cdot L_{slab} \cdot w_{Df} + 0.438 \cdot w_{Lf} \cdot L_{slab}$ $\quad R_f = 20.6\,m \frac{kN}{m}$

Loading on edge beam:

Beam self weight: $w_{Db} := \left[b_w \cdot (h_b - h_s)\right] \cdot (\gamma_c \cdot g)$ $\quad w_{Db} = 1.47 \frac{kN}{m}$ $\quad \alpha_D \cdot w_{Db} = 1.84 \frac{kN}{m}$

$w_f := \alpha_D \cdot w_{Db} + \frac{R_f}{b_{slab}}$ $\quad w_f = 22.4 \frac{kN}{m}$

Positive bending moment at midspan: $L_b := l_{nb} + b_{sup}$ $\quad L_b = 7900\,mm$

$M_f := \frac{1}{8} \cdot w_f \cdot L_b^2$ $\quad M_f = 174.84\,kN \cdot m$

Normalized moment: Assume $d_b := 30 \cdot mm$ $\quad cc := 30 \cdot mm$ $\quad d_{st} := 10 \cdot mm$

$d := h_b - cc - d_{st} - \frac{d_b}{2}$ $\quad d = 395\,mm$

$K_r := \frac{M_f}{b_{eff} \cdot d^2}$ $\quad K_r = 1.28\,MPa$ Note: Use b_{eff}

$$\rho := \frac{\left[\phi_c \cdot \alpha_1 \cdot f'_c - \left(\phi_c^{\,2} \cdot \alpha_1^{\,2} \cdot f'_c{}^2 - 2 \cdot K_r \cdot \phi_c \cdot \alpha_1 \cdot f'_c\right)^{\left(\frac{1}{2}\right)}\right]}{(f_y \cdot \phi_s)} \qquad \rho = 0.00395$$

$A_s := \rho \cdot b_{eff} \cdot d$ $\quad A_s = 1365.1\,mm^2$ <--- Governs

$A_{s\_min} := \frac{0.2 \cdot \sqrt{f'_c \cdot MPa}}{f_y} \cdot b_w \cdot h_b$ $\quad A_{s\_min} = 308.1\,mm^2$ $\quad b_w = 250\,mm$ $\quad h_b = 450\,mm$

Say use 3 No. 25 bars $\quad A_{bar} := 500 \cdot mm^2$ $\quad n_{bar} := 3$ $\quad d_b := 25 \cdot mm$

$A_s := n_{bar} \cdot A_{bar}$ $\quad A_s = 1500\,mm^2$

Check: $\Sigma F_x = 0$ $\quad C_c = T$

$(\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (a \cdot b) = \phi_s \cdot f_y \cdot A_s$ $\quad a := \phi_s \cdot f_y \cdot \frac{A_s}{(\phi_c \cdot \alpha_1 \cdot f'_c \cdot b)}$ $\quad a = 117.3\,mm$ < 200 mm OK

$c := \frac{a}{\beta_1}$ $\quad c = 131.1\,mm$

$\varepsilon_s := \varepsilon_{cu} \cdot \left(\frac{d - c}{c}\right)$ $\quad \varepsilon_s = 0.00705$ $\quad \frac{\varepsilon_s}{\varepsilon_y} = 3.52$ OK

Check beam spacing:

Bar spacing: $agg := 20 \cdot mm$ - Aggregate size $cc = 30\,mm$

$$
s := \left|
\begin{array}{l}
s_1 \leftarrow 1.4 \cdot d_b \\
s_2 \leftarrow 1.4 \cdot agg \\
s_3 \leftarrow s_1 \ \text{ if } \ s_1 \ge s_2 \\
s_3 \leftarrow s_2 \ \text{ otherwise} \\
s_4 \leftarrow s_3 \ \text{ if } \ s_3 \ge 30 \cdot mm \\
s_4 \leftarrow 30 \cdot mm \ \text{ otherwise} \\
s_4
\end{array}
\right.
\qquad s = 35\,mm
$$

Check beam web width: $b_{min} := 2 \cdot (cc + 10 \cdot mm) + n_{bar} \cdot d_b + (n_{bar} - 1) \cdot s$

$b_{min} = 225\,mm$ $< b_w = 250$ mm - OK